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Brief note

MODELLING ACCURACY OF A CAR STEERING MECHANISM WITH RACK AND PINION AND MCPHERSON SUSPENSION

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Modelling accuracy of a car steering mechanism with a rack and pinion and McPherson suspension is analyzed. Geometrical parameters of the model are described by using the coordinates of centers of spherical joints, directional unit vectors and axis points of revolute, cylindrical and prismatic joints. Modelling accuracy is assumed as the differences between the values of the wheel knuckle position and orientation coordinates obtained using a simulation model and the corresponding measured values. The sensitivity analysis of the parameters on the model accuracy is illustrated by two numerical examples.

Key words: steering mechanism, suspension, model, accuracy.

1. Introduction

The steering mechanism with rack and pinion and McPherson suspension is the most common system used in passenger cars. Although, the planar model of steering linkage has received a lot of attention for the purpose of minimizing steering errors, no attempt has been made so far to investigate the sensitivities of spatial orientations of the wheel knuckle relative to variation of geometrical parameters (Meissonier *et al.* [1]). The objective of the proposed optimization is to minimize the maximum steering error during cornering. This is followed by a sensitivity analysis to predict how the steering error is affected by manufacturing tolerances and assembly errors. Since the optimized error is very sensitive to the variations of the parameters, the sensitivity optimization of the steering linkage is performed in an integrated manner.

A typical independent-type McPherson strut suspension system of Fig.1 consists of a revolute joint at point A_1 and spherical joints at points A_2, A_3, B_1, B_3, and cylindrical joint at point B_6, respectively. Link A_1B_1 represents the lower arm connected as a revolute joint with the chassis and as a spherical joint with the wheel assembly. Link A_3B_3 represents the tie rod connected as a spherical joint with the chassis and the wheel assembly. Also, link A_2B_6 represents the strut connected with the spring-damper and as a cylindrical joint with the wheel assembly and as a spherical joint with the chassis. Therefore, this McPherson strut suspension can be modeled as a platform parallel mechanism with three legs: 1. Revolute-Spherical (R-S) link for lower arm modeling, 2. Spherical-Spherical (S-S) link for tie- rod modeling and 3. Spherical-Cylindrical (S-C) link for strut modeling.

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Vehicle wheels are installed to the chassis frame with geometrically appropriate angles and distances considering the drivability, stability and steerability. Those geometrical factors related to the wheel positions are called the static design factors which are important to be determined at an early design stage since they determine the dynamic characteristics of the vehicles. Those are caster angle, camber angle, toe angle, and kingpin inclination angle.

The static design factors of the suspension system are: the wheel camber angle (γ), wheel steer angle (δ), caster angle (τ) and kingpin inclination angle (σ) of the steering axis. Figure 2 shows the variations of the wheel steer angle during the bump/rebound motion, respectively. Similarly, the variations of kingpin inclination angle and caster angle can be obtained.

In this paper, a sensitivity analysis for the kinematic static design factor determining the motion characteristics of suspension systems and a sensitivity analysis and optimization are carried out with which the designers can consider the riding quality and steering stability in suspension system design and predict the change of suspension factors required depending on the vehicle characteristics. This may help the designers to determine layout of the suspension system and to develop the integrated optimum design system of suspension (Mantaras *et al*. [2]).

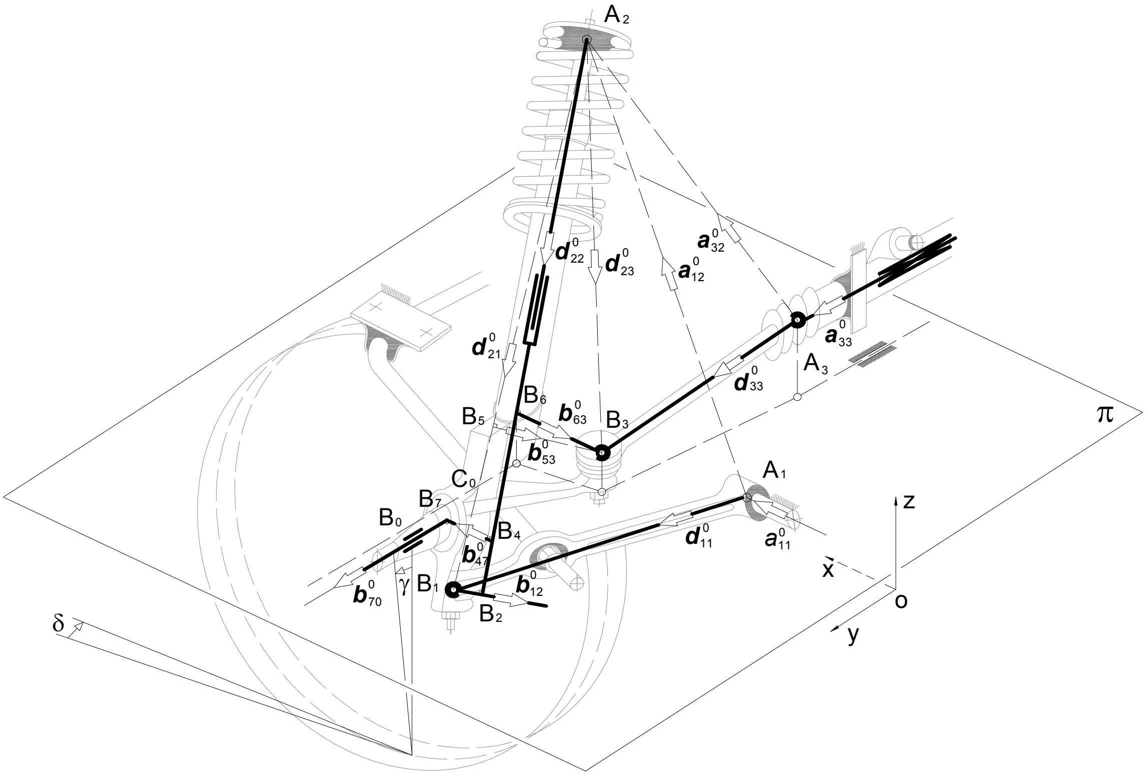

Fig.1. Steering mechanism with rack and pinion and McPherson suspension.

2. Evaluation of the modelling accuracy

The inaccuracy in mechanism modelling arises because of dimensional errors due to manufacturing, elastic deflections in assembling and caused by external load. For the evaluation of the modelling accuracy, the first–order differential equation is assumed, by inspecting the errors in the position and orientation of the knuckle coordinate frame due to dimensional errors.

$$\Delta \boldsymbol{b}(s,p) = \sum \left(\frac{\partial \boldsymbol{b}}{\partial l_i} \Delta l_i + \frac{\partial \boldsymbol{b}}{\partial \alpha_i} \Delta \alpha_i \right) ,$$

$\boldsymbol{b} = \left[b_x \; b_y \; b_z \; \delta_x \;\; \delta_y \;\; \delta_z \right]^T$ - vector of coordinates describing the position and orientation of the knuckle coordinate frame with respect to the body frame;
$\boldsymbol{l} = \left[l_1 \; ... \; l_n \right]^T$, $\boldsymbol{\alpha} = [\alpha_1 \; ... \; \alpha_n]^T$ – vectors of linear and angular dimensions of the mechanism;
Δl_i, $\Delta\alpha_i$ – tolerances of the dimensions under consideration;
s – variable used to describe the strut position;
p – variable describing the rack displacement, proportional to the angle of the steering wheel.

Sensitivity analysis renders it possible to predict how the steering error is affected by manufacturing tolerances and assembly errors. The steering error is very sensitive to the variations of the parameters. The steering error is determined as the difference between the nominal value of the steer angle ($\delta_{N(L/R)}$) obtained by using formulae (1- 23) with the nominal values of mechanism dimensions (given by the producer) and the simulated value ($\delta_{p(L/R)}$) obtained with changed (by ± 2%) values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63},d_{33}) with respect to their nominal values.

$$\Delta_\delta = \delta_{N(L/R)} - \delta_{p\pm 2(L/R)},$$

$$\Delta_\gamma = \gamma_{N(L/R)} - \gamma_{p\pm 2(L/R)}.$$

Numerical Example 1

The dimensions of the mechanism under consideration:
d_{12}*=566.8*; d_{11}*= 248.6*; b_{07}*=97.0*; b_{63}*=122.6*; d_{33}*=255.6*; b_{12}*=18.2*; b_{13}*=221.7*; b_{26}*=183.2*; b_{24}*= 76.7*; $\boldsymbol{a}_2$(*-25.6*; *463.0*; *75.,0*); $\boldsymbol{a}_3$(*-87.6*; *264.0*; *364.0*); $\boldsymbol{a}_1$(*11.8*; *330.6*; *207.2*)[*mm*]; α_1*=100.63°*; α_2*= 96.13°*; l_b*=1289*; l_d*=256*; l_z*=528* [*mm*].

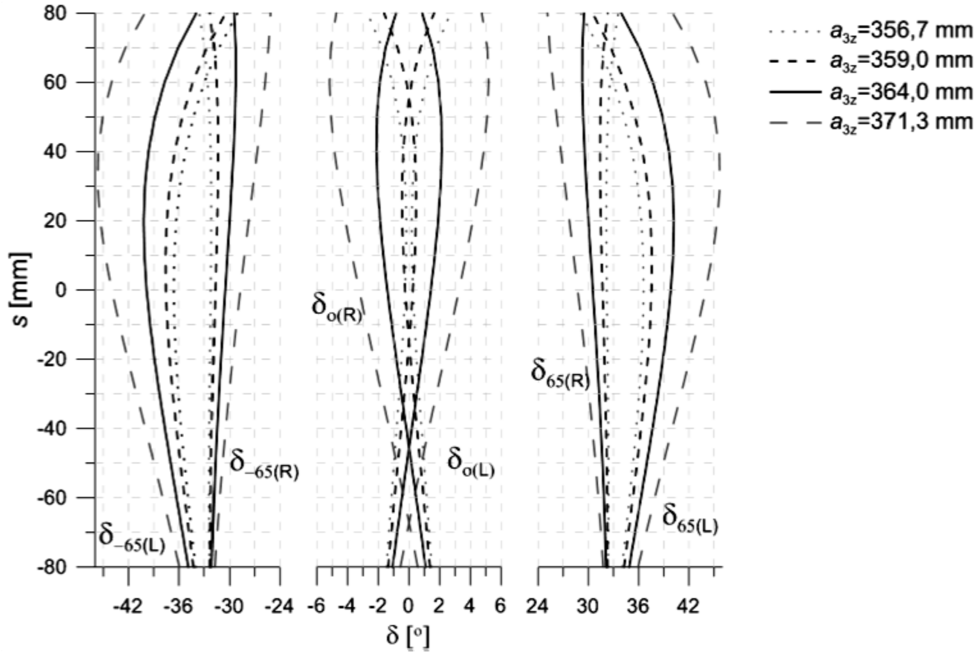

Fig.2. The influence of a_{3z} on the steer angle δ versus the displacement of the strut s, calculated by using the simulation model for: straight ahead position, where δ_{oR} – the steer angle of the right wheel, δ_{oL} – left wheel; for the displacement of the rack (*p=65*), where $\delta_{R,65}$ – right wheel, $\delta_{L,65}$- left wheel.

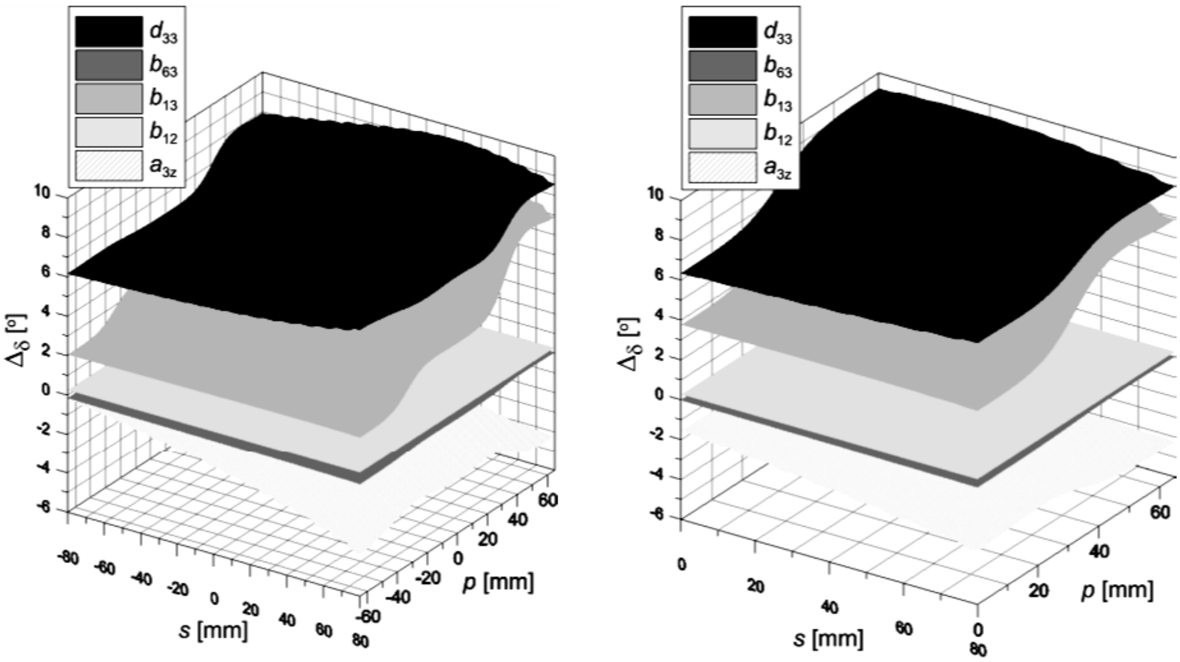

Fig.3 Surfaces of variations of the steering error determined for the assumed values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63},d_{33}) different (by ± *2*%) from the respective nominal values.

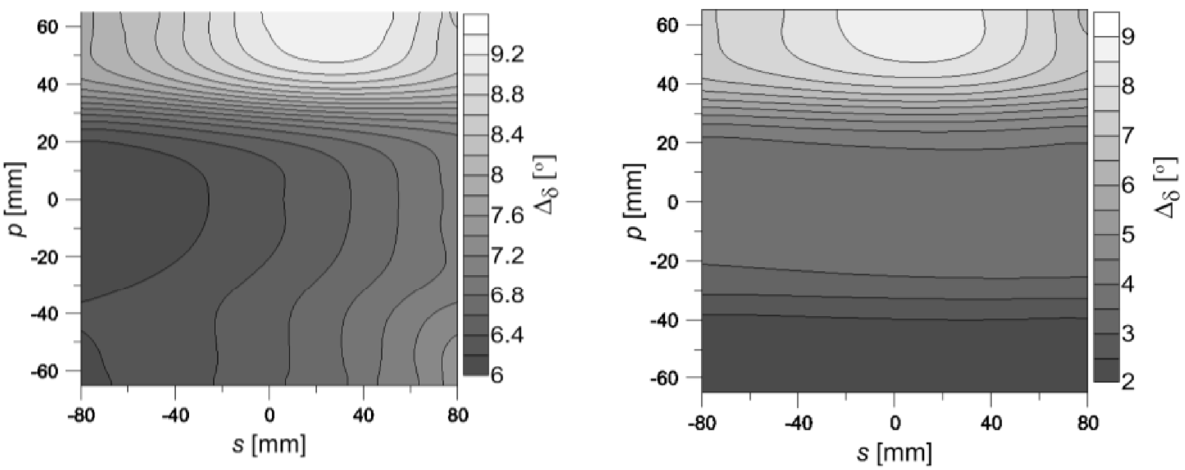

Fig.4. The sections of surfaces of variations of the steering error determined for the assumed values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63}, d_{33}) different (by ± *2*%) from the respective nominal values.

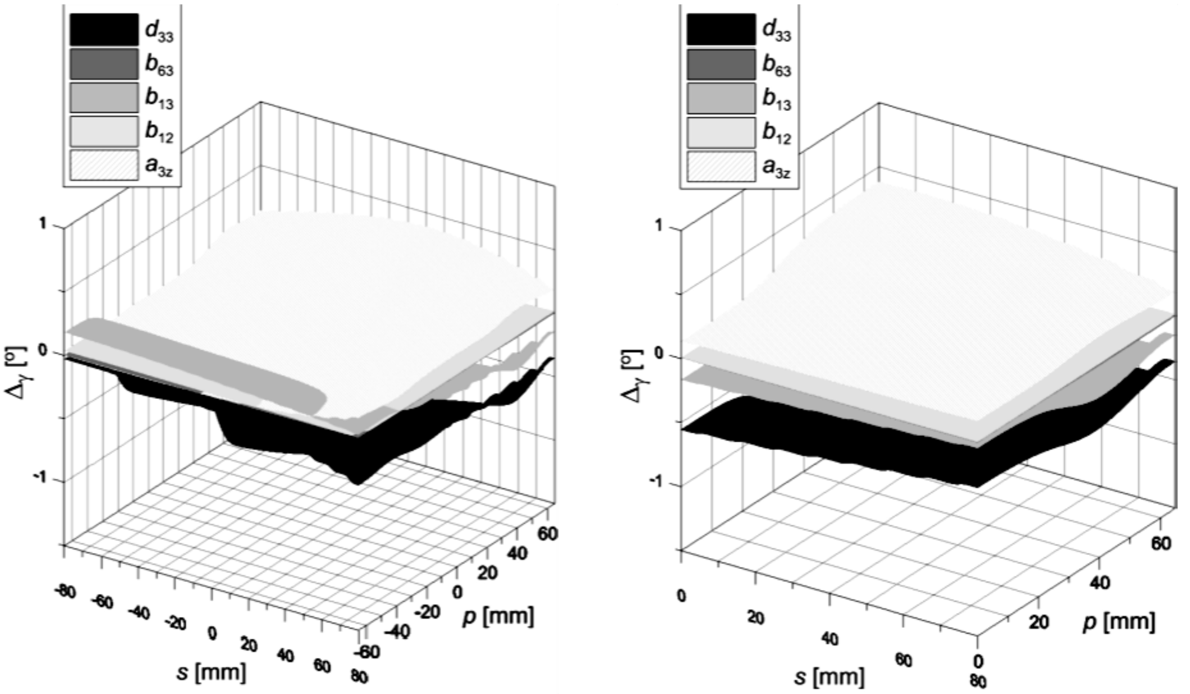

Fig.5. Surfaces of variations of the camber angle error determined for the assumed values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63}, d_{33}) different (by ± 2%) from the respective nominal values.

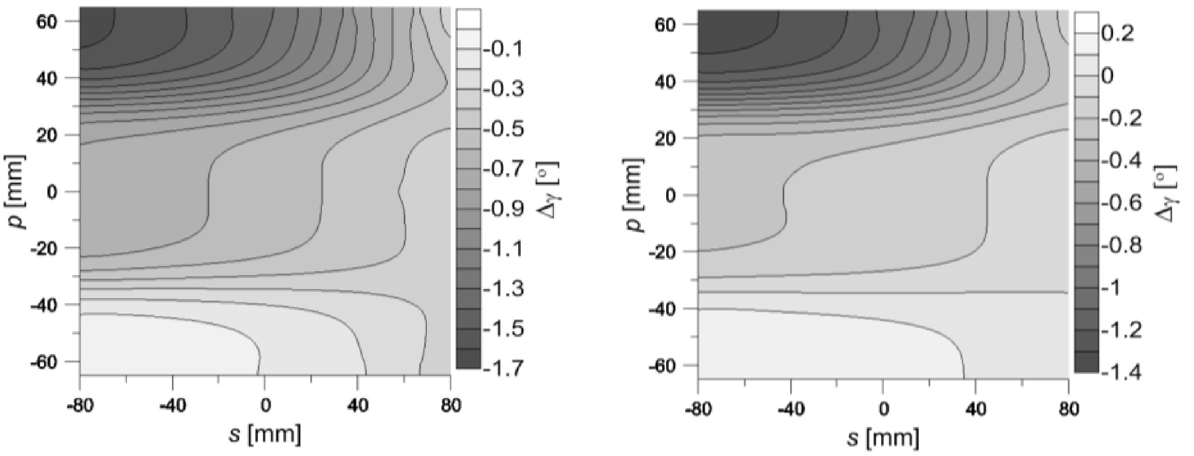

Fig.6. The sections of surfaces of variations of the camber angle error determined for the assumed values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63},d_{33}) different (by ± 2%) from the respective nominal values.

Numerical Example 2

d_{11}= 395.8; b_{63}=126.0; d_{33}=435.7; b_{12}=57.3; b_{13}=124.4; b_{26}=51.5; a_{2x}=271.9; a_{2y}= -504.5; a_{2z}= 616.7; a_{3x}= 147.0; a_{3y}=-239.9; a_{3z}= 51.0; a_{1x}=245.7; a_{1y}= -308.5; a_{1z}=22.0 *[mm]*, $\alpha_1 = \arc\cos\left(-\boldsymbol{b}^o_{70} \cdot \boldsymbol{d}^o_{22}\right) = 81.5^o$; $\alpha_2 = \arc\cos\left(\boldsymbol{b}^o_{70} \cdot \boldsymbol{b}^o_{63}\right) = 98.5^o$;

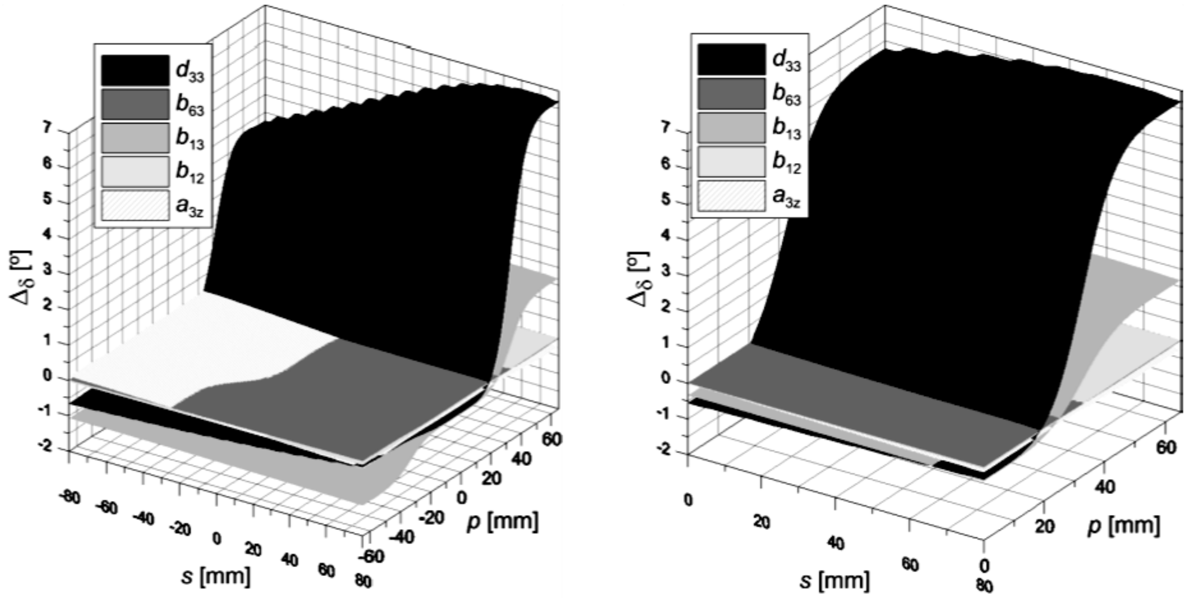

Fig.7. Surfaces of variations of the steering error determined for the assumed values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63}, d_{33}) different (by ± 2%) from the respective nominal values.

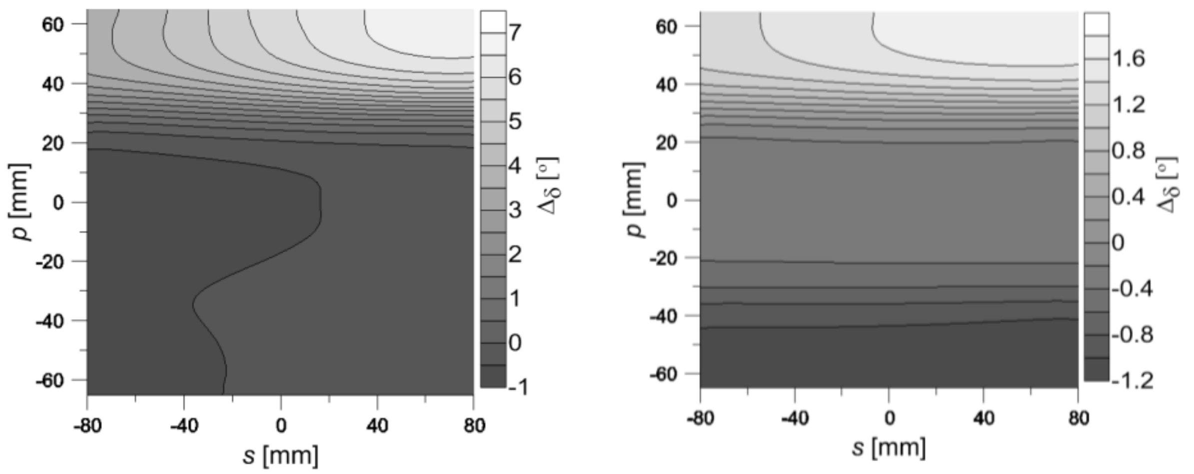

Fig.8. The sections of surfaces of variations of the steering error determined for the assumed values of the dimensions (a_{3z}, b_{12}, b_{13}, b_{63},d_{33}) different (by ± 2%) from the respective nominal values.

3. Formulary used for simulations

$$c_{12} = \boldsymbol{a}_{12}^{0} \cdot \boldsymbol{a}_{11}^{0}, \tag{3.1}$$

$$c_{1} = \left(d_{11}^{2} + a_{12}^{2} - b_{12}^{2} - s^{2} \right) \Big/ 2d_{11} \cdot a_{12}, \tag{3.2}$$

$$D_{1} = 1 - c_{12}^{2} - c_{1}^{2}, \tag{3.3}$$

$$\boldsymbol{d}_{11}^{0} = \left[\boldsymbol{a}_{12}^{0} \cdot c_1 - \boldsymbol{a}_{11}^{0} \cdot c_{12} \cdot c_1 - \left(\boldsymbol{a}_{12}^{0} \times \boldsymbol{a}_{11}^{0} \right) \cdot \sqrt{D_1} \right] \Big/ \left(1 - c_{12}^2 \right), \tag{3.4}$$

$$\boldsymbol{d}_{21}^{0} = \left(\boldsymbol{d}_{11}^{0} \cdot d_{11} - \boldsymbol{a}_{12}^{0} \cdot a_{12} \right) \Big/ \sqrt{s^2 + b_{12}^2}\,, \tag{3.5}$$

$$c_3 = \left[s^2 + b_{12}^2 + \left(s - b_{26} \right)^2 + b_{63}^2 - b_{13}^2 \right] \Big/ \left(2 \sqrt{\left[\left(s - b_{26} \right)^2 + b_{63}^2 \right] \cdot \left(s^2 + b_{12}^2 \right)} \right) \tag{3.6}$$

$$c_4 = \left[\left(s - b_{26} \right)^2 + b_{63}^2 + a_{32}^2 - d_{33}^2 \right] \Big/ \left(2 \cdot a_{32} \cdot \sqrt{\left[\left(s - b_{26} \right)^2 + b_{63}^2 \right]} \right), \tag{3.7}$$

$$c_5 = -\boldsymbol{d}_{21}^{0} \cdot \boldsymbol{a}_{32}^{0}\,, \tag{3.8}$$

$$D_2 = 1 - c_3^2 - c_4^2 - c_5^2 + 2 \cdot c_3 \cdot c_4 \cdot c_5\,, \tag{3.9}$$

$$\boldsymbol{d}_{23}^{0} = \left[+\boldsymbol{d}_{21}^{0} \cdot \left(c_3 - c_4 \cdot c_5 \right) - \boldsymbol{a}_{32}^{0} \cdot \left(c_4 - c_3 \cdot c_5 \right) + \left(+\boldsymbol{d}_{21}^{0} \times \boldsymbol{a}_{32}^{0} \right) \cdot \sqrt{D_2} \right] \Big/ \left(1 - c_5^2 \right), \tag{3.10}$$

$$c_6 = s \Big/ \sqrt{s^2 + b_{12}^2}\,, \tag{3.11}$$

$$c_7 = \left(s - b_{26} \right) \Big/ \sqrt{\left(s - b_{26} \right)^2 + b_{63}^2}\,, \tag{3.12}$$

$$c_8 = \boldsymbol{d}_{23}^{0} \cdot \boldsymbol{d}_{21}^{0}\,, \tag{3.13}$$

$$D_3 = 1 - c_6^2 - c_7^2 - c_8^2 + 2 \cdot c_6 \cdot c_7 \cdot c_8\,, \tag{3.14}$$

$$\boldsymbol{d}_{22}^{0} = \left[+\boldsymbol{d}_{21}^{0} \cdot \left(c_6 - c_7 \cdot c_8 \right) + \boldsymbol{d}_{23}^{0} \cdot \left(c_7 - c_6 \cdot c_8 \right) + \left(\boldsymbol{d}_{21}^{0} \times \boldsymbol{d}_{23}^{0} \right) \cdot \sqrt{D_3} \right] \Big/ \left(1 - c_8^2 \right), \tag{3.15}$$

$$\boldsymbol{b}_{63}^{\mathrm{o}} = \left[\boldsymbol{d}_{23}^{\mathrm{o}} \sqrt{\left(s - b_{26} \right)^2 + b_{63}^2} - \boldsymbol{d}_{22}^{0} \left(s - b_{26} \right) \right] \Big/ b_{63}\,, \tag{3.16}$$

$$\boldsymbol{b}_{70}^{\mathrm{o}} = \left[-\boldsymbol{d}_{22}^{\mathrm{o}} c_9 + \boldsymbol{b}_{63}^{\mathrm{o}} c_{10} + \left(-\boldsymbol{d}_{22}^{\mathrm{o}} \times \boldsymbol{b}_{63}^{\mathrm{o}} \right) \cdot \sqrt{D_4} \right] \Big/ \left(1 - c_{11}^2 \right), \tag{3.17}$$

$$c_9 = \boldsymbol{d}_{22}^{\mathrm{o}} \cdot \boldsymbol{b}_{70}^{\mathrm{o}} = \cos\alpha_1\,, \qquad c_{10} = \boldsymbol{b}_{70}^{0} \cdot \boldsymbol{b}_{63}^{\mathrm{o}} = \cos\alpha_2\,, \qquad c_{11} = \boldsymbol{d}_{22}^{\mathrm{o}} \cdot \boldsymbol{b}_{63}^{0}\,, \tag{3.18}$$

$$D_4 = 1 - c_9^2 - c_{10}^2\,, \tag{3.19}$$

$$\delta = \operatorname{arctg}\left(\frac{\boldsymbol{b}_{70\mathrm{x}}^{\mathrm{o}}}{\boldsymbol{b}_{70\mathrm{y}}^{\mathrm{o}}} \right), \tag{3.20}$$

$$\gamma = \mathrm{arctg}\left(\frac{\boldsymbol{b}^{\mathrm{o}}_{70\mathrm{z}}}{\sqrt{\left(\boldsymbol{b}^{\mathrm{o}}_{70\mathrm{x}}\right)^{2}+\left(\boldsymbol{b}^{\mathrm{o}}_{70\mathrm{y}}\right)^{2}}}\right), \tag{3.21}$$

$$\sigma = \arccos\left(c_{9}\right) = \mathrm{arctg}\left(\frac{-\boldsymbol{d}^{\mathrm{o}}_{21\mathrm{y}}}{\boldsymbol{d}^{\mathrm{o}}_{21\mathrm{z}}}\right), \tag{3.22}$$

$$\tau = \arccos\left(c_{10}\right) = \mathrm{arctg}\left(\frac{-\boldsymbol{d}^{\mathrm{o}}_{21\mathrm{x}}}{\boldsymbol{d}^{\mathrm{o}}_{21\mathrm{z}}}\right). \tag{3.23}$$

Conclusions

Real values of the geometrical dimensions can be identified by the estimation procedure (Knapczyk and Para [3]; Knapczyk and Maniowski [4, 5, 6]) and next used in simulation. The matrix of the influence coefficients can be used to accelerate the estimation procedure.

The results of the modelling accuracy of the mechanism being under consideration indicate that it can demonstrate selected dimensions with the biggest influence on the changes of the orientation angles of the wheel knuckle during vertical displacements.

An optimal design of McPherson strut suspension system has been studied. Also, sensitivity analyses for the kinematic static design factor and for reaction forces at joints were carried out, from which the effects of each hard point(design variable) on suspension factors can be found. These studies may be effectively applied to determine the suspension system layout by predicting the variations of suspension factors required for vehicle characteristics at early design stages. The method employed can be extended to develop the integrated suspension design system.

Nomenclature

Input variables: p - displacement of the rack; s- length of the spring-damper module.
Output variables: δ - wheel steer angle; γ - wheel camber angle; τ - caster angle; σ - kingpin inclination angle.

A_i – centre of the joint connected the link i to the car body

B_i – centre of the joint connected the link i to the wheel knuckle

$\boldsymbol{a}_i = \begin{bmatrix} a_{ix} & a_{iy} & a_{iz} \end{bmatrix}^T$ – position vector of point A_i described in the body reference frame

$\boldsymbol{b}_i = \begin{bmatrix} b_{ix} & b_{iy} & b_{iz} \end{bmatrix}^T$ – position vector of point B_i described in the body reference frame

$\boldsymbol{a}_{ij}, \boldsymbol{b}_{ij}, \boldsymbol{d}_{ij}$ – position vectors: $\boldsymbol{a}_{ij} = A_i A_j$, $\boldsymbol{b}_{ij} = B_i B_j$, $\boldsymbol{d}_{ij} = B_i A_j$

$\boldsymbol{a}^{0}_{ij}, \boldsymbol{b}^{0}_{ij}, \boldsymbol{d}^{0}_{ij}, \boldsymbol{e}^{0}_{ij}$ – unit vectors of the position vectors

d_{ii} – i-th link length ($i = 1, 3$)

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